Webs and alternating sign matrices

Lecture 2: Webs and tableaux
and trip = prom

CMND Summer School

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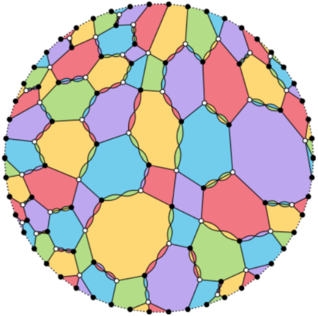
North Dakota State University

June 2026

Webs in Algebra, Geometry, Topology and Combinatorics



Dec 8 - 12, 2025

Topical Workshop

ICERM

A polynomial basis

Given the polynomial ring $\mathbb{C}[x]$, its basis is $\{1, x, x^2, \ldots\}$.

$$(3x-1)(2x+5) = 6x^2 + 13x - 5$$

An SL_2-invariant polynomial

This is an SL_2-invariant polynomial: $x_1 y_2 - x_2 y_1$

An SL_2-invariant polynomial

This is an SL_2-invariant polynomial: $x_1y_2 - x_2y_1$

Here's why: If you apply a change of variables by multiplying your matrix of variables by a 2×2 determinant 1 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} = \begin{pmatrix} ax_1 + by_1 & ax_2 + by_2 \\ cx_1 + dy_1 & cx_2 + dy_2 \end{pmatrix}$$

and then write your polynomial under the change of variables

$$\begin{aligned} &(ax_1 + by_1)(cx_2 + dy_2) - (ax_2 + by_2)(cx_1 + dy_1) \\ = &\, acx_1x_2 + adx_1y_2 + bcx_2y_1 + bdy_1y_2 - acx_1x_2 - adx_2y_1 - bcx_1y_2 - bdy \\ = &\, ad(x_1y_2 - x_2y_1) + bc(x_2y_1 - x_1y_2) = (ad - bc)(x_1y_2 - x_2y_1) \\ = &\, 1 \cdot (x_1y_2 - x_2y_1) \end{aligned}$$

you get the same polynomial back.

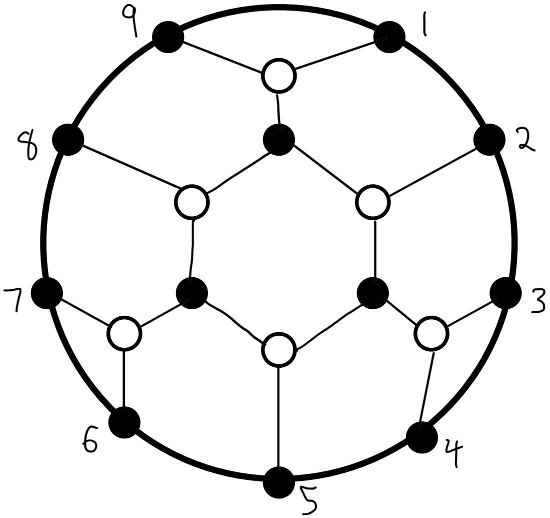
An SL_3-invariant polynomial

$$\det \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}$$
$$= x_1y_2z_3 + x_2y_3z_1 + x_3y_1z_2 - x_3y_2z_1 - x_1y_3z_2 - x_2y_1z_3$$

Another SL_3-invariant polynomial

$$\det\begin{pmatrix}
1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&x_8&0\\
0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&y_8&0\\
0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&z_8&0\\
1&0&0&1&0&0&0&0&0&0&x_2&0&0&0&0&0&0&0\\
0&1&0&0&1&0&0&0&0&0&y_2&0&0&0&0&0&0&0\\
0&0&1&0&0&1&0&0&0&0&z_2&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&x_5&0&0&0&0\\
0&0&0&0&1&0&0&1&0&0&0&0&0&y_5&0&0&0&0\\
0&0&0&0&0&1&0&0&1&0&0&0&0&z_5&0&0&0&0\\
1&0&0&0&0&0&0&0&0&x_1&0&0&0&0&0&0&0&x_9\\
0&1&0&0&0&0&0&0&0&y_1&0&0&0&0&0&0&0&y_9\\
0&0&1&0&0&0&0&0&0&z_1&0&0&0&0&0&0&0&z_9\\
0&0&0&1&0&0&0&0&0&0&0&x_3&x_4&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&y_3&y_4&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&z_3&z_4&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&x_6&x_7&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&y_6&y_7&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&z_6&z_7&0&0
\end{pmatrix}$$

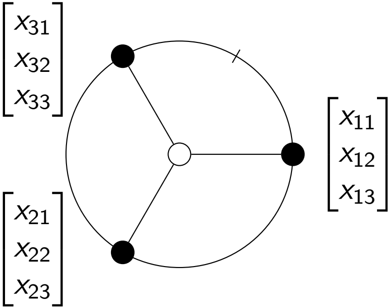
An SL_3-invariant polynomial as a web



Webs and web invariants

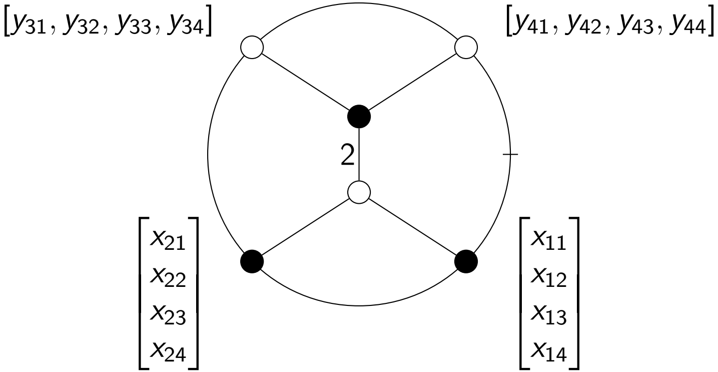
Definition (convention of Fraser, Lam, Le 2019)

An SL_r-**tensor diagram** is a properly bicolored edge-weighted graph embedded in a disk with labeled boundary vertices and whose interior vertices have degree r. A tensor diagram is a **web** if it is planar.

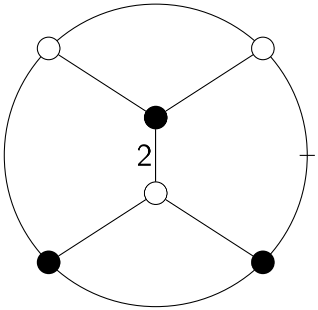


$$\det \begin{pmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \\ x_{13} & x_{23} & x_{33} \end{pmatrix}$$

An SL_4 web and invariant (up to sign, see Fomin-Pylyvaskyy 2016)



An SL_4 web and invariant (up to sign, see Fomin-Pylyvaskyy 2016)



$$\det \begin{pmatrix} 1 & 0 & 0 & 0 & x_{11} & x_{21} \\ 0 & 1 & 0 & 0 & x_{12} & x_{22} \\ 0 & 0 & 1 & 0 & x_{13} & x_{23} \\ 0 & 0 & 0 & 1 & x_{14} & x_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} & 0 & 0 \\ y_{41} & y_{42} & y_{43} & y_{44} & 0 & 0 \end{pmatrix}$$

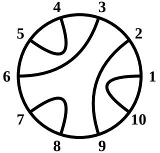
An SL_r-tensor diagram with boundary type $\underline{c}$ represents an SL_r-invariant polynomial in $\mathrm{Inv}_{U_q(\mathfrak{sl}_r)}(\bigwedge_q^{\underline{c}} V_q)$.

SL_r-webs span this space. Which ones give a basis?

Rotation-invariant web bases for SL_2 and SL_3

Theorem (Temperley-Lieb 1971)

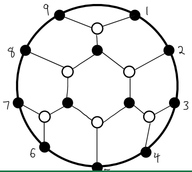
Noncrossing matchings of $2c$ give a rotation-invariant basis for SL_2.

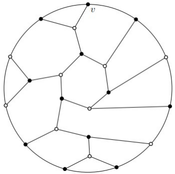


$$\begin{vmatrix} x_{11} & x_{110} \\ x_{21} & x_{210} \end{vmatrix} \begin{vmatrix} x_{12} & x_{19} \\ x_{22} & x_{29} \end{vmatrix} \begin{vmatrix} x_{13} & x_{16} \\ x_{23} & x_{26} \end{vmatrix} \begin{vmatrix} x_{14} & x_{15} \\ x_{24} & x_{25} \end{vmatrix} \begin{vmatrix} x_{17} & x_{18} \\ x_{27} & x_{28} \end{vmatrix}$$

Theorem (Kuperberg 1996, Khovanov and Kuperberg 1999)

Non-elliptic (no cycles of length less than 6) SL_3 webs with $3c$ boundary vertices give a rotation-invariant basis for SL_3.



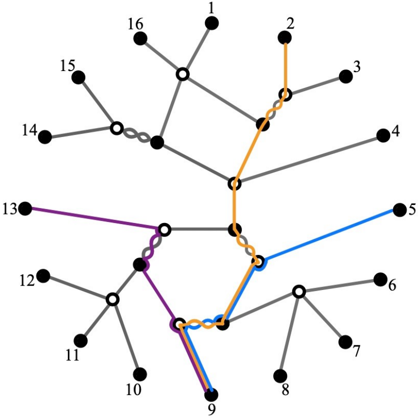


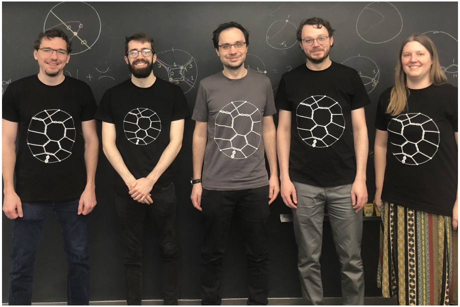
"The main problem [. . .] is how to generalize them to higher rank." (Kuperberg '96)

Rotation-invariant web bases for SL_4

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

Contracted, fully-reduced, top 4-hourglass plabic graphs with $4c$ *boundary vertices give a rotation-invariant web basis for* SL_4.



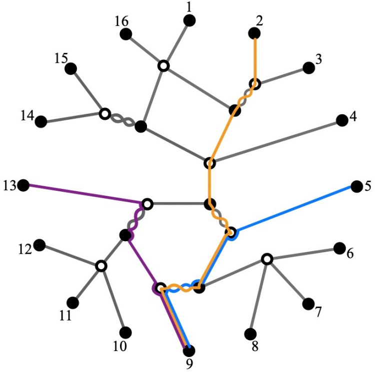


Hourglass plabic graphs

Definition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

An r-**hourglass plabic graph** is an SL_r web with edges of weight $k > 1$ drawn as twisted hourglasses.

A 4-hourglass plabic graph of boundary type $\underline{c} = (1, 1, \ldots, 1)$

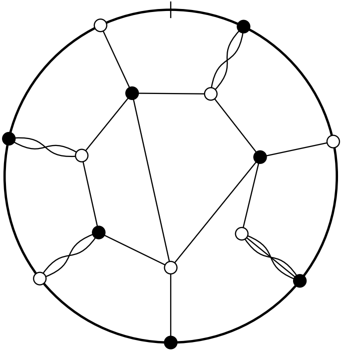


Hourglass plabic graphs

Definition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

An *r*-**hourglass plabic graph** is an SL_r web with edges of weight $k > 1$ drawn as twisted hourglasses.

A 4-hourglass plabic graph of boundary type $\underline{c} = (2, \bar{1}, 3, 1, \bar{2}, 2, \bar{1})$

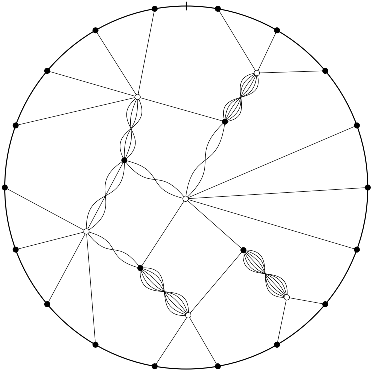


Hourglass plabic graphs

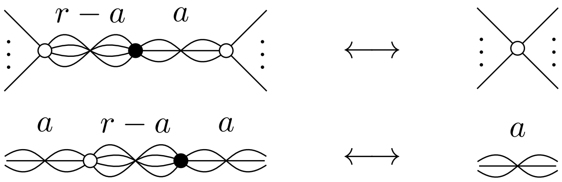
Definition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

An r-**hourglass plabic graph** is an SL_r web with edges of weight $k > 1$ drawn as twisted hourglasses.

A 9-hourglass plabic graph of boundary type $\underline{c} = (1, 1, \ldots, 1)$

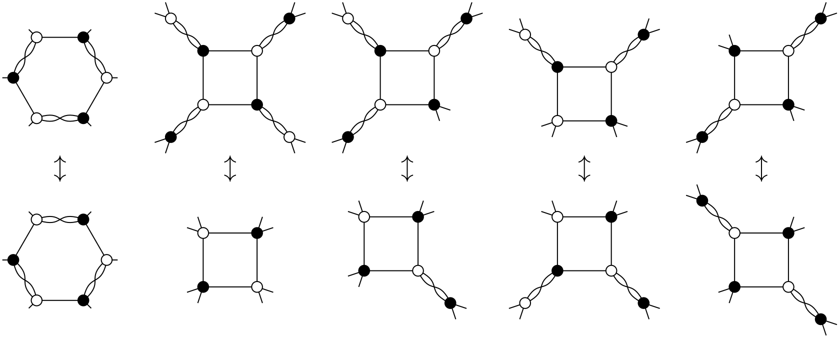


Contraction moves



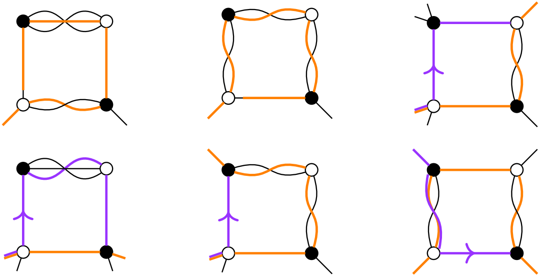
Contracted means apply the above moves as much as possible.

Benzene and square moves for $r = 4$



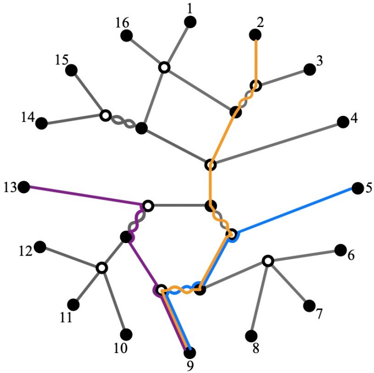
- Square moves do not change the polynomial invariant, so it doesn't matter which representative you choose.
- Top means do as many benzene moves as possible to get to your prefered of the two configurations.

Fully reduced

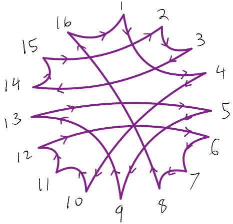


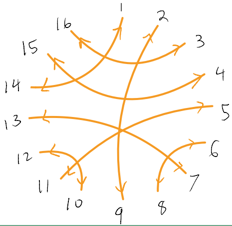
- Fully reduced for $r = 4$ means no graph in the move equivalence class has a 4-cycle containing an hourglass.
- For general r, fully reduced includes that the sum of the edge multiplicities around a square must be at most r.

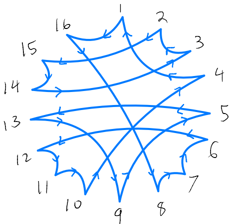
(Hourglass) plabic graphs have trip permutation(s)



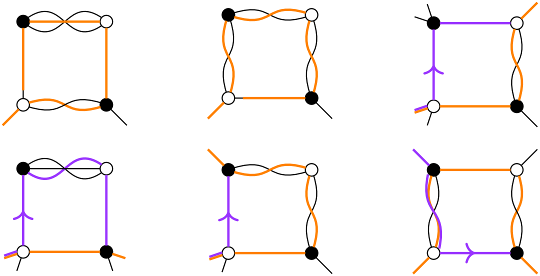
The **trip$_i$ permutation** of an hourglass plabic graph is a map on boundary vertices created by taking the ith left at each ○ and the ith right at each ●.





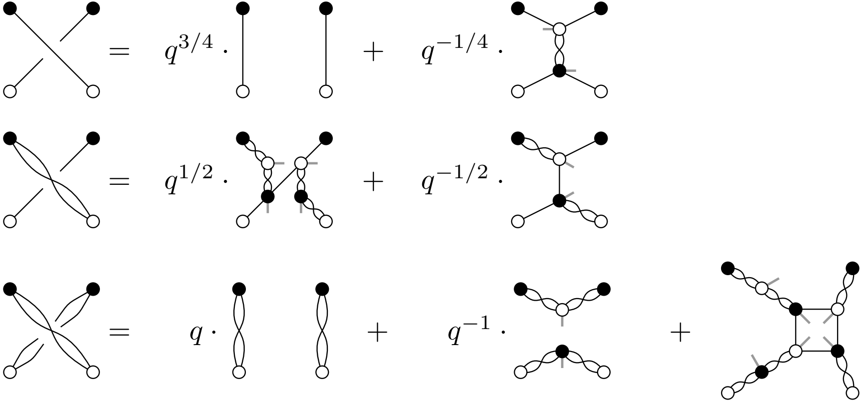


Trip theoretic fully reduced

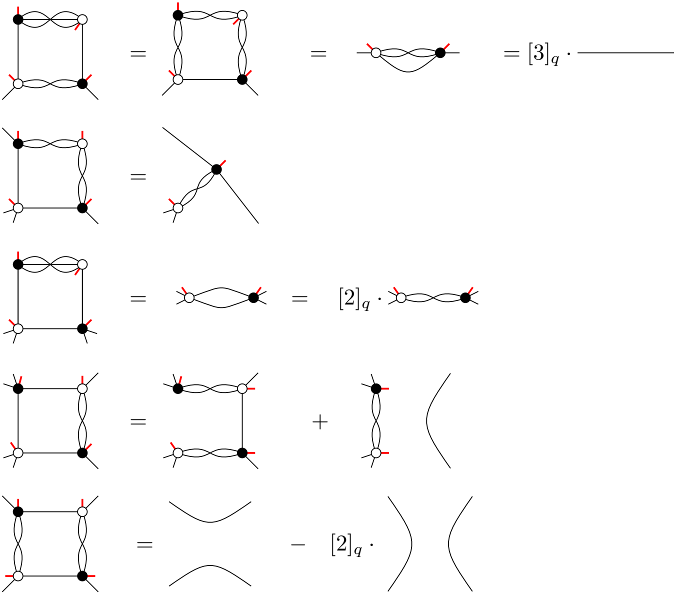


- Alternatively, fully reduced means the trips are *monotonic*, meaning they have no *bad double crossings*: no interior isolated components, no self-intersections, no two trip$_i$-segments have an oriented double crossing, and no pair of trip$_i$- and trip$_{i+1}$-segments have an oriented double crossing.

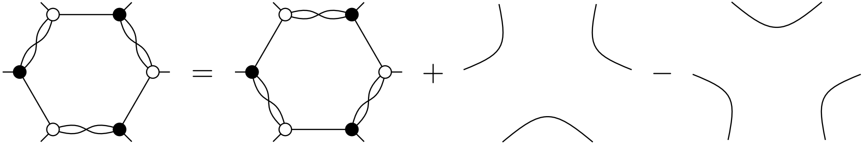
Skein relations: Uncrossing relations



Skein relations: Decomposing forbidden 4-cycles



Skein relations: The benzene relation



Tableaux

Definition

A **standard Young tableau** is a filling of a partition shape λ with the numbers $1, 2, \ldots, n$ (where $|\lambda| = n$) such that the rows and columns are increasing and each number is used once.

$$T = \emptyset \to 1 \to 2 \to 21 \to 31 \to 32 \to 33 \to 43 \to 53 \to 63 \to 631 \to 632$$

1	2	4	7	8	9
3	5	6			
10	11				

$\longleftrightarrow$ Yamanouchi lattice word:
1 1 2 1 2 2 1 1 1 3 3

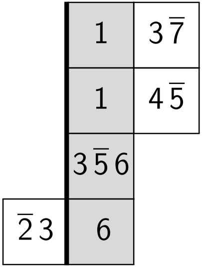
A lattice word with alphabet $\{1, 2, \ldots\}$ is **Yamanouchi** if the number of i's stays ahead of the number of $i+1$'s.

Fluctuating tableaux

Definition

An r-**fluctuating tableau** $T = \lambda^0 \xrightarrow{c_1} \lambda^1 \xrightarrow{c_2} \cdots \xrightarrow{c_n} \lambda^n$ is a sequence of r-row generalized partitions such that λ^{i-1} and λ^i differ by c_i.

$$T = 0000 \xrightarrow{2} 1100 \xrightarrow{\overline{1}} 110\overline{1} \xrightarrow{3} 2110 \xrightarrow{1} 2210 \xrightarrow{\overline{2}} 2100 \xrightarrow{2} 2111 \xrightarrow{\overline{1}} 1111$$



$\longleftrightarrow$

Yamanouchi lattice word:
$\{1,2\}\ \overline{4}\ \{1,3,4\}\ 2\ \{\overline{2},\overline{3}\}\ \{3,4\}\ \overline{1}$

A lattice word with alphabet subsets of $\{1,\overline{1},2,\overline{2},\ldots\}$ is **Yamanouchi** if the number of i's minus the number of $\overline{i}$'s stays ahead of the number of $(i+1)$'s minus the number of $\overline{i+1}$'s.

Tableaux give a non rotation-invariant basis

Standard monomial theory (Hodge 1943, Seshadri 1978...) gives a tableau basis for SL_r-invariants.

(It's relatively easy to work with, but it's not rotation-invariant.)

So the number of basis elements of any basis equals the number of tableaux.


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Gromotion on tableaux via jeu de taquin

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

Gromotion on tableaux via jeu de taquin

	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

Gromotion on tableaux via jeu de taquin

2		5	6
3	4	7	10
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3		7	10
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2	4	5	6
3	7		10
8	9	11	14
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2	4	5	6
3	7	10	
8	9	11	14
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2	4	5	6
3	7	10	14
8	9	11	
12	13	15	16

Gromotion on tableaux via jeu de taquin

2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	

Gromotion on tableaux via jeu de taquin

2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	**1**

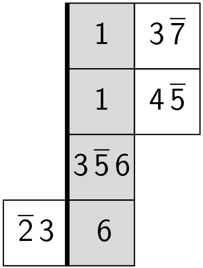
Gromotion on tableaux via jeu de taquin

1	3	4	5
2	6	9	13
7	8	10	15
11	12	14	16

To instead computer **promotion**, subtract 1 (mod n).

Promotion on rectangular tableaux behaves like rotation

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16



Theorem (Haiman 1992; Rhoades 2010 cyclic sieving)

For any $a \times b$ standard Young tableau T, $\mathrm{Pro}^{ab}(T) = T$.

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2024)

For any rectangular fluctuating tableau T of length n, $\mathrm{Pro}^{n}(T) = T$.

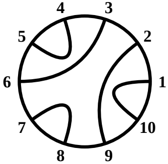
Promotion on standard Young tableaux

Theorem (D. White)

Promotion on $2 \times c$ standard Young tableaux $\leftrightarrow$ rotation on non-crossing matchings of $2c$.

1	2	3	4	7
5	6	8	9	10

$\longleftrightarrow$

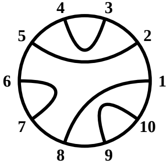


Promotion

Rotation

1	2	3	6	9
4	5	7	8	10

$\longleftrightarrow$



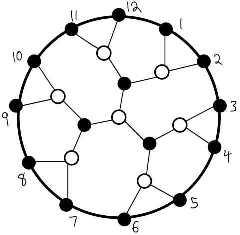
Promotion on standard Young tableaux

Theorem (Petersen, Pylyavskyy, and Rhoades 2009)

Promotion on $3 \times c$ standard Young tableaux $\leftrightarrow$ web rotation.

1	3	5	7
2	4	9	11
6	8	10	12

$\longleftrightarrow$



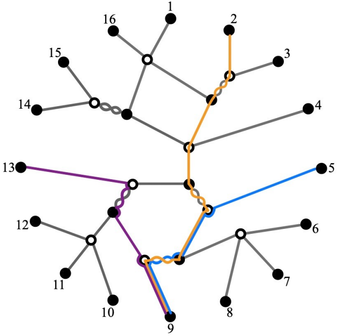
Rotation-invariant web bases for SL_4

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

- *Contracted, fully-reduced, top 4-hourglass plabic graphs with $4c$ boundary vertices give a rotation-invariant web basis for SL_4.*
- *Promotion on $4 \times c$ standard Young tableaux $\leftrightarrow$ rotation of hourglass plabic graph equivalence classes.*

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longleftrightarrow$



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Nathan Williams (University of Texas at Dallas)

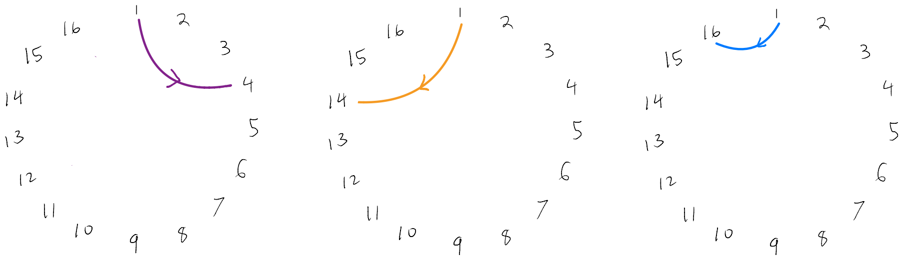
Promotion permutations (Hopkins-Rubey(i=1)2022, GPPSS 2024)

Define the digraph $\mathrm{prom}_i(T)$ by drawing $y \to x$ if x moves between rows $i+1$ and i in the yth application of gromotion.

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longrightarrow$

2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	**1**



Promotion permutations (Hopkins-Rubey(i=1)2022, GPPSS 2024)

Define the digraph $\mathrm{prom}_i(T)$ by drawing $y \to x$ if x moves between rows $i+1$ and i in the yth application of gromotion.

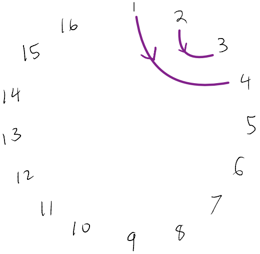
1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

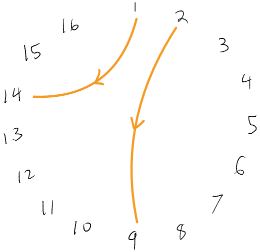
$\longrightarrow$

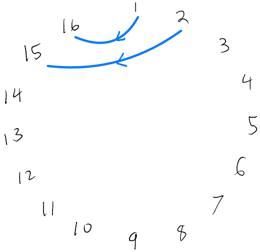
2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	**1**

$\longrightarrow$

3	4	5	6
7	9	10	14
8	11	15	16
12	13	**1**	**2**







Promotion permutations (Hopkins-Rubey(i=1)2022, GPPSS 2024)

Define the digraph $\mathrm{prom}_i(T)$ by drawing $y \to x$ if x moves between rows $i+1$ and i in the yth application of gromotion.

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longrightarrow$

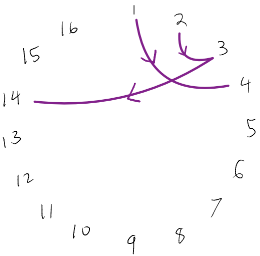
2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	**1**

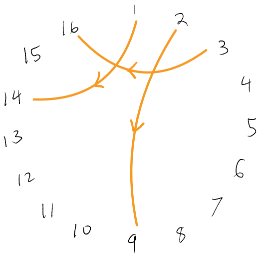
$\longrightarrow$

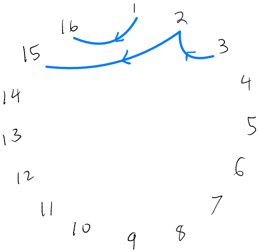
3	4	5	6
7	9	10	14
8	11	15	16
12	13	**1**	**2**

$\longrightarrow$

4	5	6	14
7	9	10	16
8	11	15	**2**
12	13	**1**	**3**







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1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\rightarrow$

2	4	5	6
3	7	10	14
8	9	11	16
12	13	15	**1**

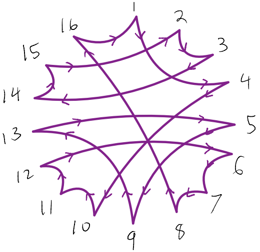
$\rightarrow$

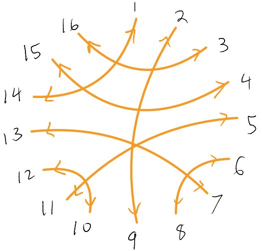
3	4	5	6
7	9	10	14
8	11	15	16
12	13	**1**	**2**

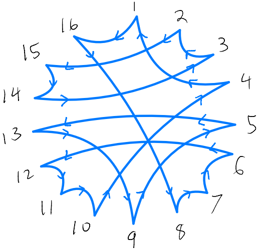
$\rightarrow$

4	5	6	14
7	9	10	16
8	11	15	**2**
12	13	**1**	**3**

$\rightarrow$







Promotion permutations

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2024)

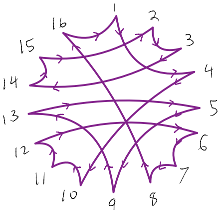
Given an $r \times c$ standard Young tableau T, its promotion digraphs are permutations, rotate, and satisfy $\mathrm{prom}_i(T) = \mathrm{prom}_{r-i}(T)^{-1}$.
The antiexcedances of $\mathrm{prom}_i(T)$ are exactly the numbers in the first i rows of T.

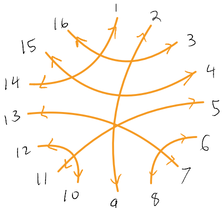
1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

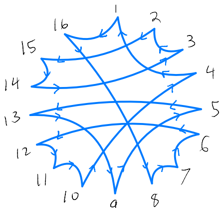
$\mathrm{prom}_1(T) = 4\ 3\ 14\ 10\ 9\ 7\ 8\ 16\ 13\ 11\ 12\ 6\ 5\ 15\ 2\ 1$

$\mathrm{prom}_2(T) = 14\ 9\ 16\ 15\ 11\ 8\ 13\ 6\ 2\ 12\ 5\ 10\ 7\ 1\ 4\ 3$

$\mathrm{prom}_3(T) = 16\ 15\ 2\ 1\ 13\ 12\ 6\ 7\ 5\ 4\ 10\ 11\ 9\ 3\ 14\ 8$







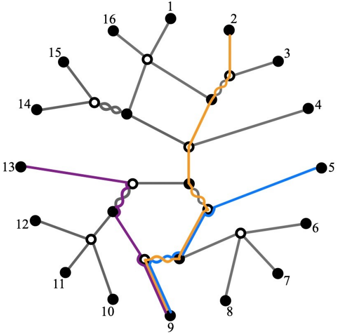
Rotation-invariant web bases for SL_4

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

- *Contracted, fully-reduced, top 4-hourglass plabic graphs with* $4c$ *boundary vertices give a rotation-invariant web basis for* SL_4.
- *Promotion on* $4 \times c$ *standard Young tableaux* $\leftrightarrow$ *rotation of hourglass plabic graph equivalence classes.*
- *For* $i = 1, 2, 3$, $\mathrm{trip}_i(G) = \mathrm{prom}_i(T)$.

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

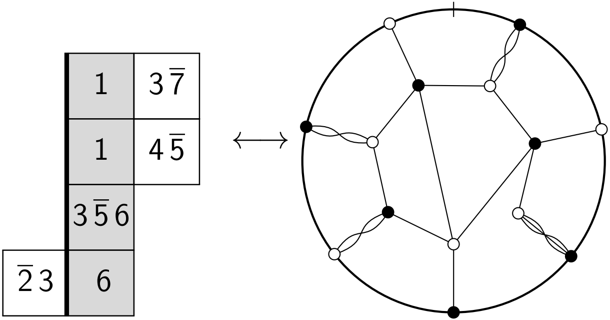
$\longleftrightarrow$



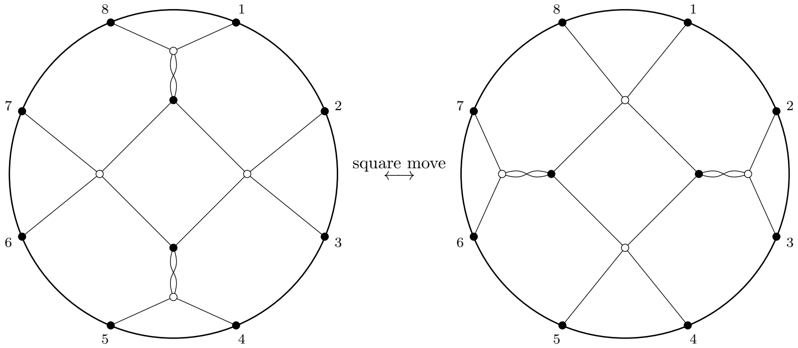
Rotation-invariant web bases for SL_4

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2023+)

- *Contracted, fully-reduced, top 4-hourglass plabic graphs with* $4c$ *boundary vertices give a rotation-invariant web basis for* SL_4.
- *Promotion on* 4 *row fluctuating tableaux* $\leftrightarrow$ *rotation of hourglass plabic graph equivalence classes.*
- *For* $i = 1, 2, 3$, $\mathrm{trip}_i(G) = \mathrm{prom}_i(T)$.



Why do we need move equivalence classes?



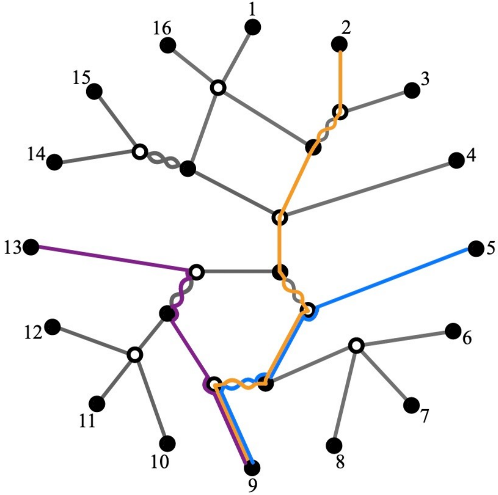
1	2
3	4
5	6
7	8

Mapping from a basis web to a tableau

The separation labeling:

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longleftarrow$

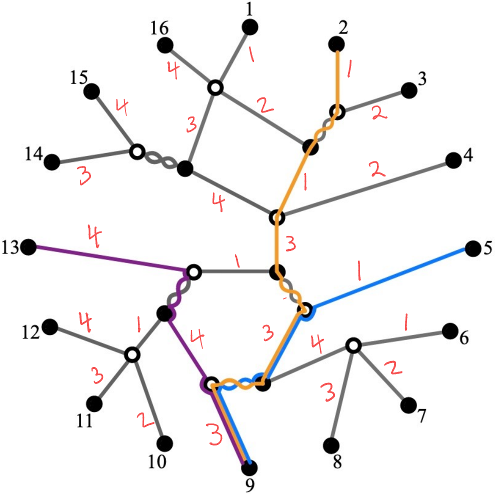


Mapping from a basis web to a tableau

The separation labeling:

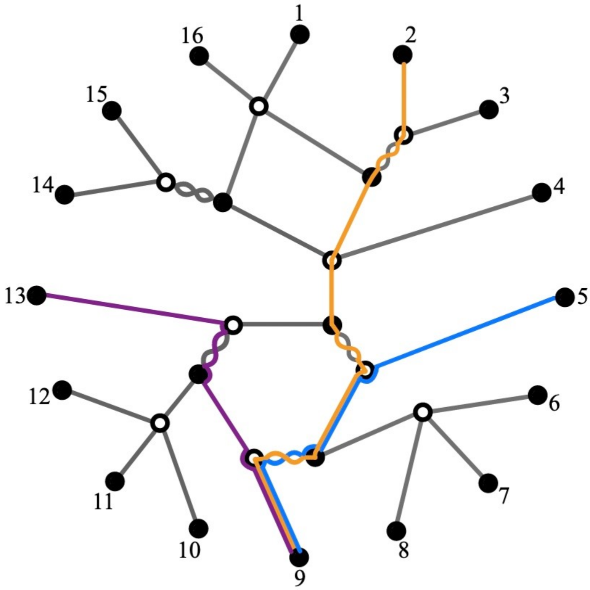
1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longleftarrow$

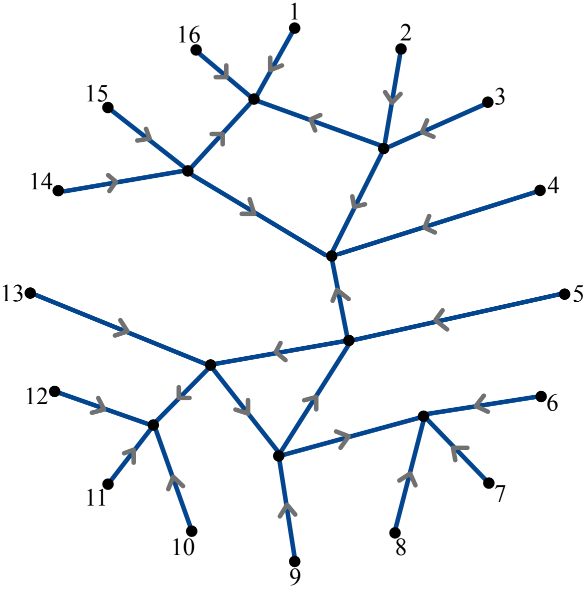


1 1 2 2 1 1 2 3 3 2 3 4 4 3 4 4

Hourglass plabic graph



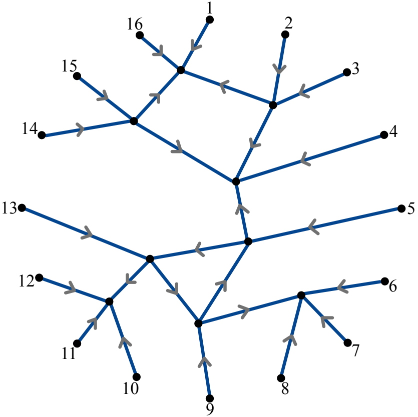
Symmetrized six-vertex configuration



Symmetrized six-vertex configuration

Definition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

A **symmetrized six-vertex configuration** is a planar graph embedded in a disk with all internal vertices of degree 4, all boundary vertices of degree 1, and each vertex a source, sink, or transmitting.



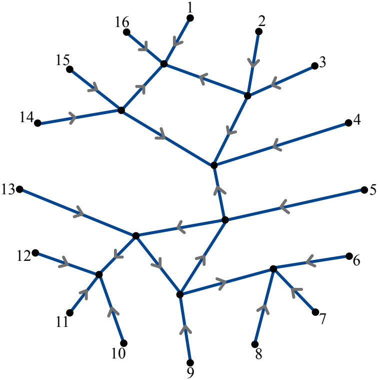
Rotation-invariant web bases for SL_4

Theorem (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

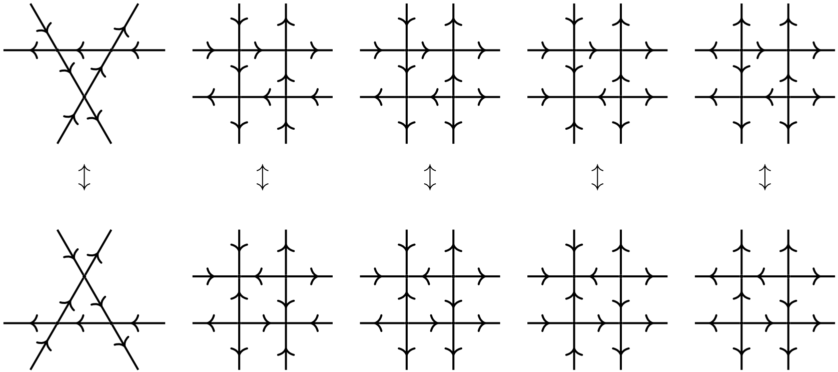
Well-oriented, top symmetrized six-vertex configurations with $4c$ *boundary vertices give a rotation-invariant basis for* SL_4*.*

1	2	5	6
3	4	7	10
8	9	11	14
12	13	15	16

$\longleftrightarrow$



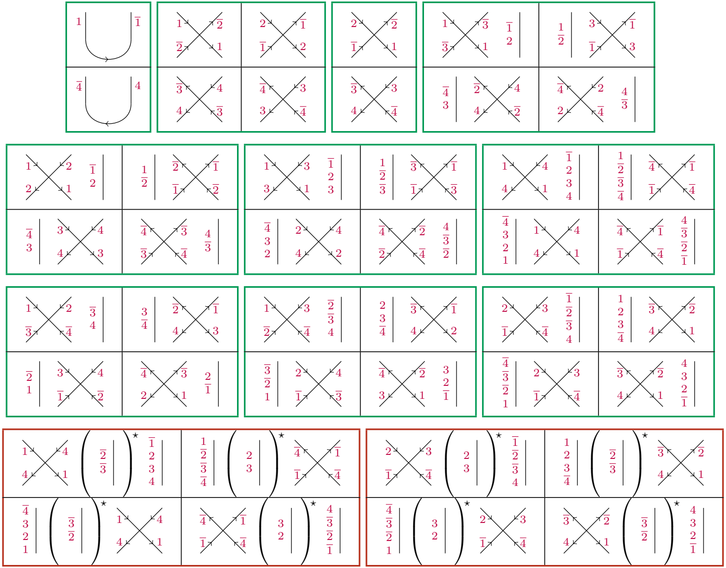
Yang-Baxter and ASM moves



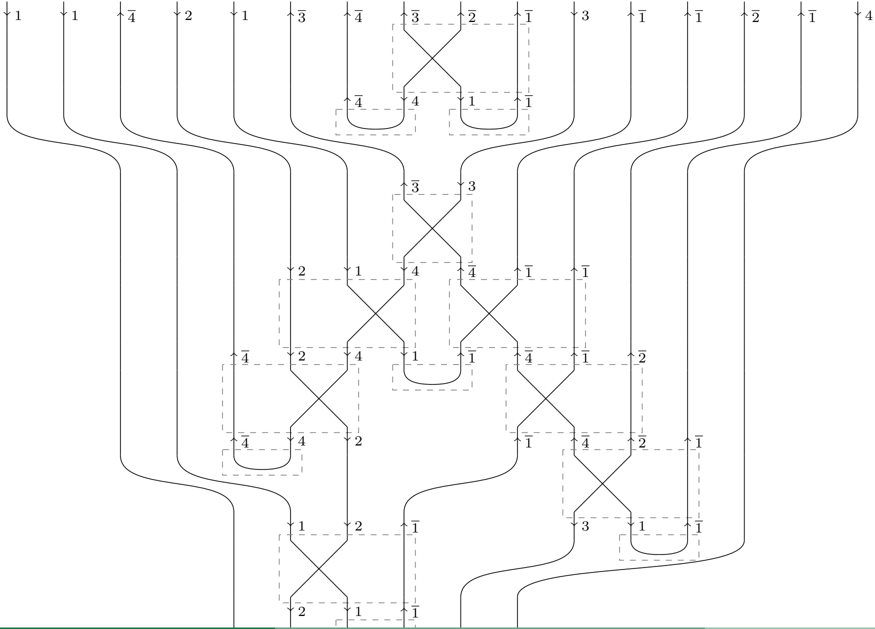
- ASM moves do not change the polynomial invariant, so it doesn't matter which representative you choose.
- Top means do as many Yang-Baxter moves as possible to get to your prefered of the two configurations.
- Well-oriented means no loops or parallel edges and every triangle in the move equivalence class is cyclically oriented.

Mapping from a tableau to a web

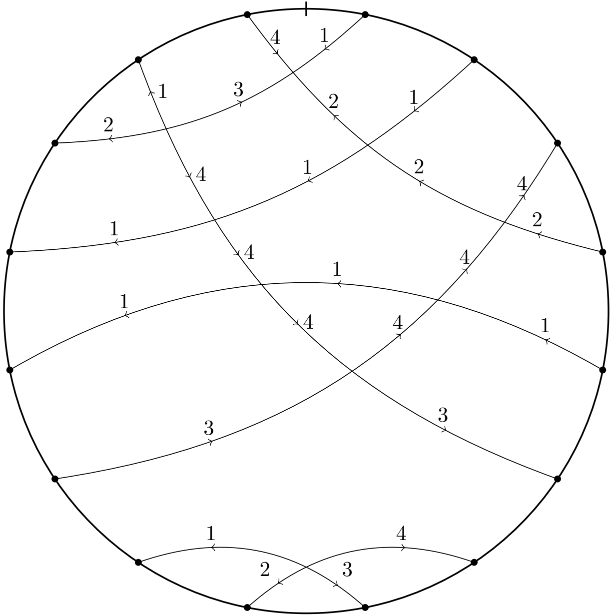
Growth rules:



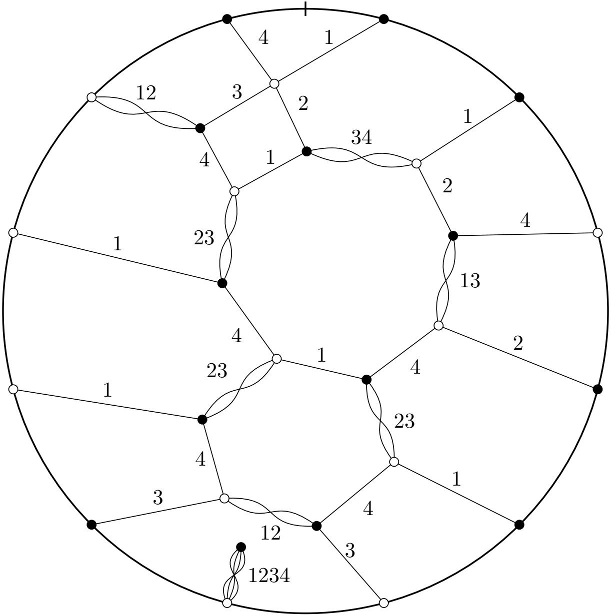
Mapping from a tableau to a web



Mapping from a tableau to a web



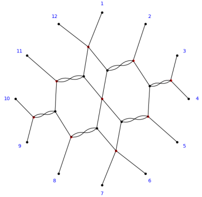
Mapping from a tableau to a web

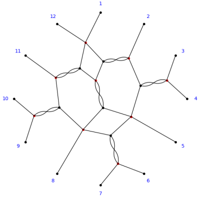


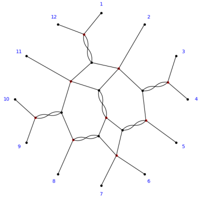
The equivalence class of the superstandard tableau

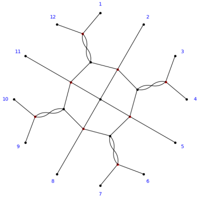
1	2	3
4	5	6
7	8	9
10	11	12

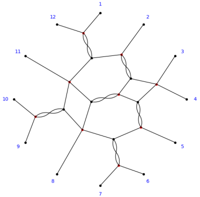
$\longleftrightarrow$

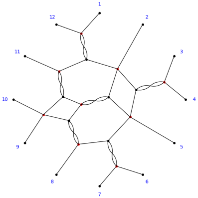


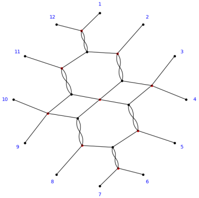












Alternating sign matrices as webs

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Proposition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

The web equivalence class of the $4 \times c$ superstandard tableau (lattice word $1^c2^c3^c4^c$) is in bijection with $c \times c$ alternating sign matrices.

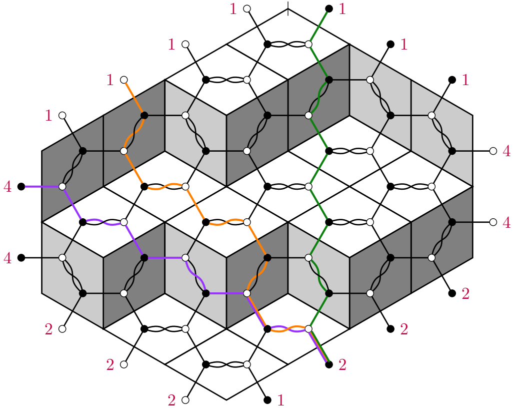
Plane partitions as webs

Proposition (Gaetz, Pechenik, Pfannerer, S., Swanson 2026)

The web equivalence class of the fluctuating tableau with lattice word $1^a\,\overline{4}^b\,2^c\,\overline{1}^{a-c}\,\overline{2}^c\,4^b\,\overline{1}^c$ *is in bijection with the set of plane partitions inside an* $a \times b \times c$ *box.*

		$1\,\overline{18}$	$2\,\overline{17}$	$3\,\overline{16}$	$9\,\overline{15}$
		$6\,\overline{12}$	$7\,\overline{11}$	$8\,\overline{10}$	
$\overline{5}\,13$	$\overline{4}\,14$				

$111\overline{4}\overline{4}2221\overline{2}\overline{2}\overline{2}44\overline{1}\overline{1}\overline{1}\overline{1}$



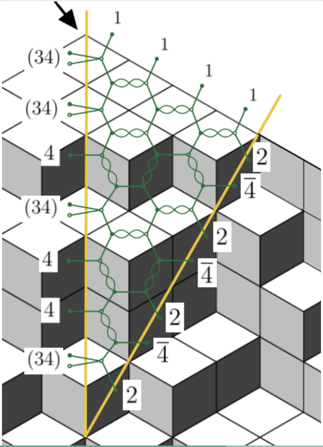
Symmetry classes of plane partitions as webs

Theorem (Adams, S. 2025+ (similar results for other symm. classes))

The web equivalence classes of fluctuating tableaux with lattice words $1^a(2\bar{4})^{a-1}2\{4^{a-1},(34)^{a-1}\}(34)$ *are in bijection with the set of totally symmetric self-complementary plane partitions in a* $2a \times 2a \times 2a$ *box.*

			1	2	3	4
			5	7	9	11
			12	16	19	21
$\overline{10}$ 13	$\overline{8}$ 14	$\overline{6}$ 15	17	18	20	22

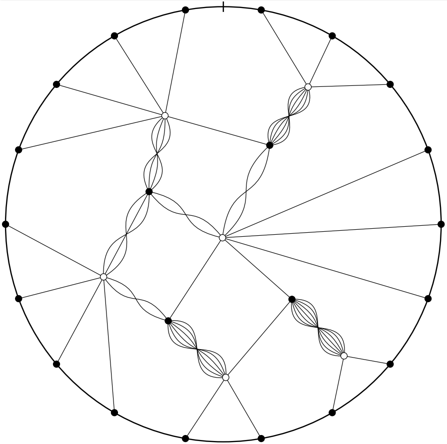
$$1^4 2\bar{4}2\bar{4}2\bar{4}2(34)4^2(34)4(34)(34)$$



Rotation-invariant web bases for SL_r in Plücker degree 2

Theorem (Fraser '23; Gaetz, Pechenik, Pfannerer, S., Swanson '25)

Contracted, fully reduced r-hourglass plabic graphs of Plücker degree two give a rotation-invariant web basis for $\mathrm{Inv}_{U_q(\mathfrak{sl}_r)}(V_q^{\otimes 2r})$.

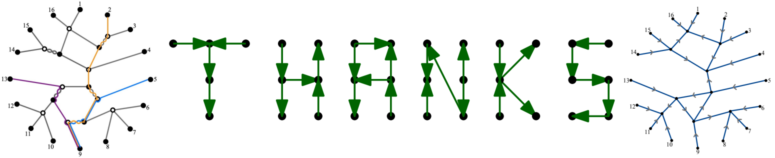


1	4
2	7
3	9
5	11
6	12
8	15
10	16
13	17
14	18

- Promotion $\leftrightarrow$ rotation

GPPSS25

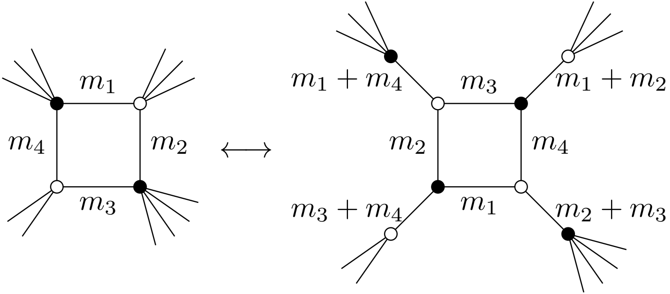
- $\mathrm{trip}_i = \mathrm{prom}_i$



- C. Gaetz, O. Pechenik, S. Pfannerer, J. Striker, and J. Swanson:
  - Rotation–invariant web bases from hourglass plabic graphs. *Inventiones Math.* **243** (2026) no.2, 102pp.
  - Web bases in degree two from hourglass plabic graphs. *International Mathematics Research Notices* **2025** (2025) no.13, 23pp.
  - Promotion permutations for tableaux. *Comb. Theory*, **4** (2024) no.2, 55pp.
- R. Patrias, O. Pechenik, and J. Striker, Promotion digraphs, `arXiv:2508.10969` (2025+)
- A. Adams and J. Striker, Webification of symmetry classes of plane partitions, `arXiv:2508.16565` (2025+)

Web bases for SL_5? and beyond?

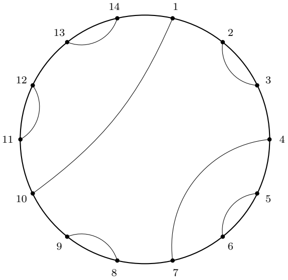
Guiding combinatorics: Promotion $\leftrightarrow$ rotation, Trip = Prom



+ benzene move analogue
+ other more complicated moves

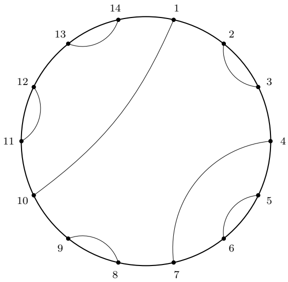
Rotation-invariant web bases for SL_r in Plücker degree 2

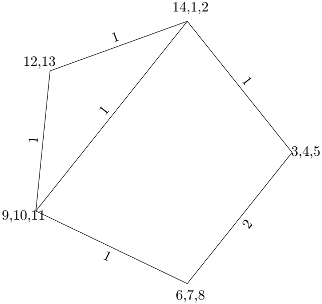
1	3
2	6
4	7
5	9
8	10
11	12
13	14



Rotation-invariant web bases for SL_r in Plücker degree 2

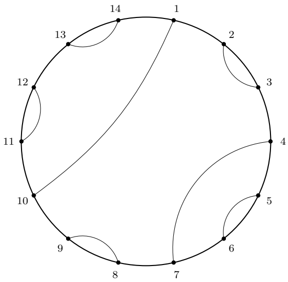
1	3
2	6
4	7
5	9
8	10
11	12
13	14

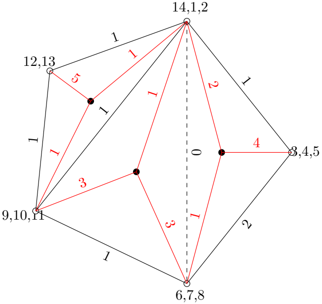




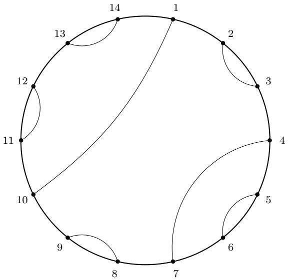
Rotation-invariant web bases for SL_r in Plücker degree 2

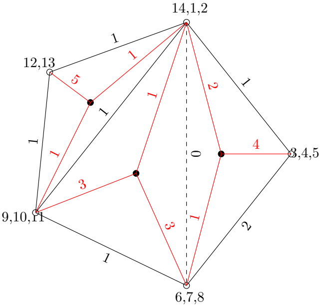
1	3
2	6
4	7
5	9
8	10
11	12
13	14

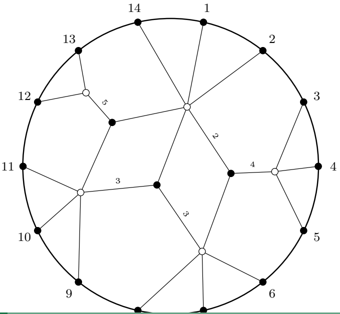




Rotation-invariant web bases for SL_r in Plücker degree 2







Rotation-invariant web bases for SL_r in Plücker degree 2